Mathematics Department – Langholm Academy

HIGHER HOMEWORK UNIT 3

CHAPTER 3

Logs and Exponentials 2

Higher - Unit 3
Logs and Exponentials 2

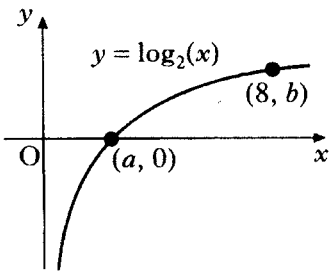
1. Evaluate x
   a. $x = \log_2 32 + \log_2 4 - \log_2 8$
   b. $4\log_x 6 + 2\log_x 4 = 1$

2. Solve to find x, giving your answer to 2 decimal places
   a. $4^x = 90$
   b. $50 = 20e^{3x}$
   c. $36 = 7^{-2x}$
   d. $15e^{-7x} = 256$

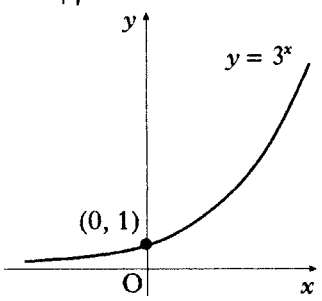
3. If $\log_a p = \cos^2 x$ and $\log_a q = \sin^2 x$ show that $pq = a$

4. Find the coordinates where the curve $y = \log_5(x-2)+1$ intersects the x axis

5. The diagram shows part of the sketch $y = \log_2 x$
   a. State the values of a and b
   b. Sketch the graph of $y = \log_2(x+2)-1$

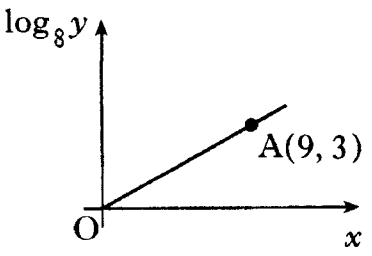


6. The diagram shows part of the graph $y = 3^x$
   a. Sketch the graph of $y = 3^{-x} - 9$
   b. Find the coordinates where it crosses the x and y axis

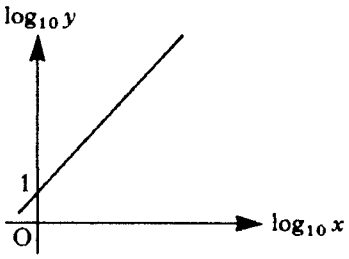


7. Functions f, g, and h are defined on suitable domains
   $f(x) = x^2 - 10x - 3$, $g(x) = 3 - x$, and $h(x) = \log_3 x$
   a) Find expressions for $h(f(x))$, and $h(g(x))$
   b) Hence solve $h(f(x)) - h(g(x)) = 2$

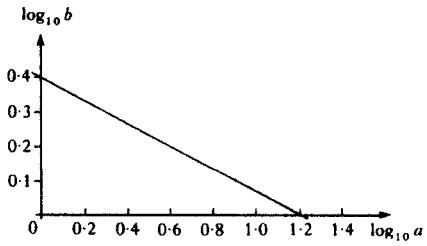
8. Two variables x and y are connected by the law $y = a^x$. The graph of $\log_8 y$ against x is a straight line passing through the origin and the point A(9,3). Find the value of a.



9. A set of experimental results give a straight line when $\log_{10} y$ is plotted against $\log_{10} x$ as shown in the diagram. The straight line passes through (0,1) and has a gradient of 2. Express y in terms of x.



10. The graph illustrates the law $b = ka^n$. Find the values of k and n.



11. A tractor tyre is inflated to a pressure of 50 units. 24 hours later the pressure has dropped to 10 units.

    a. If the pressure p_t units, after t hours is given by the formula $p_t = p_0 e^{-kt}$ find the value of k to three decimal places.
    b. The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units. If the farmer inflates the tyre to 50 units and drives the tractor for 4 hours, can the tractor be driven further without inflating the tyre and without risking further serious damage to the tyre?

12. For a particular radioactive substance, the mass m (in grams) at time t (in years) is given by

    $m = m_0 e^{-0.02t}$ where m_0 is the original mass.

    a. If the original mass is 500 grams, find the mass after 10 years
    b. The half-life of any material is the time taken for half of the mass to decay. Find the half-life of this substance.
    c. Illustrate all of the above information on a graph.